Open KIAS Winter School on Collider Physics

Feb. 13-18, 2011

Lecture 1: Basics of QCD

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Outline of the lectures

- Why QCD?
- Collider processes
- Basic properties of QCD
- Hard processes
- Fragmentation
- Hadronization and parton shower
- Parton distribution functions

Freedom and constaints

- Announced titles

  QCD, Hadronic interactions,

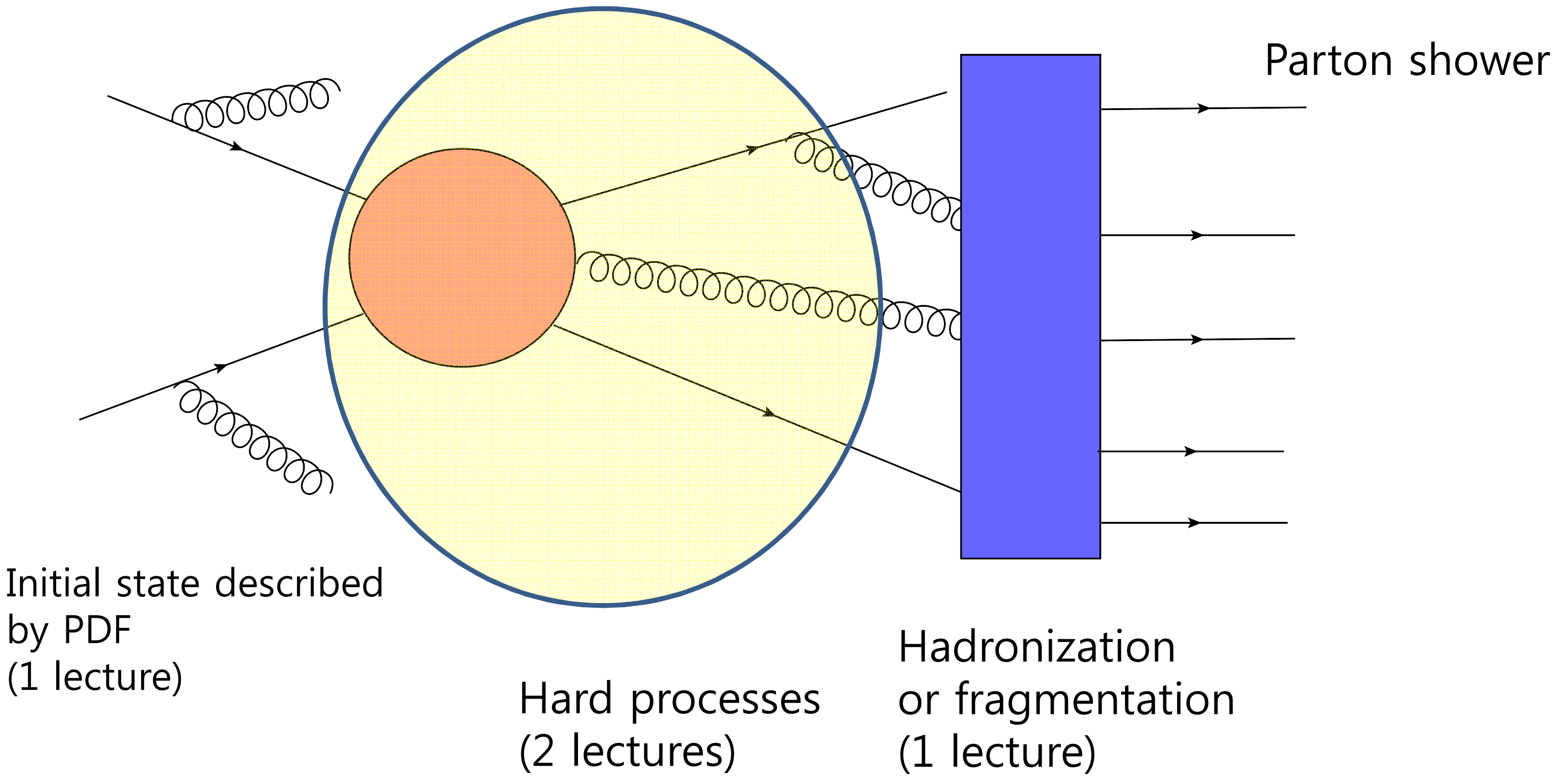
  Collider signatures of QCD, PDF and jets

  Hadronic processes: tree-level only

Standard textbooks

- Collider Physics, V. D. Barger, R. J. N. Phillips, Addison and Wesley (1997)
- Applications of Perturbative QCD, R. D. Field, Addison and Wesley (1989)
- QCD and Collider Physics, R. K. Ellis, W. J. Stirling, B. R. Webber, Cambridge (1996)
- Foundations of Quantum Chromodynamics, (3^{rd}. Ed.) T. Muta, World Scientific (2010)

Schematic hadronic processes



What we will do…

- Tree-level processes (no loops)
- Standard $2 \to 2$ processes (possibly $2 \to 3$ processes)
- Basic understanding of QCD
- We consider high-energy processes, so we neglect masses of light quarks.

Why QCD?

- Omnipresent in collider processes
- Large effects
- Enormous background for new particle search
- It appears at all scales.
- The coupling constant varies rather wildly when the scale varies.
- Sometimes we have to sum all the contributions.

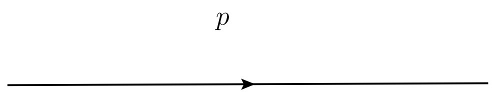
QCD

- Lagrangian

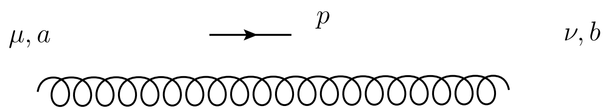
$$\mathcal{L} = \overline{\psi} i \gamma^\mu D_\mu \psi - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu a}$$

$$D^\mu = \partial_\mu - i g A^a_\mu T^a$$

- Feynman rules

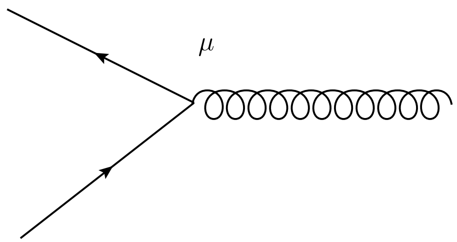


$$\frac{i}{\not{p}}$$

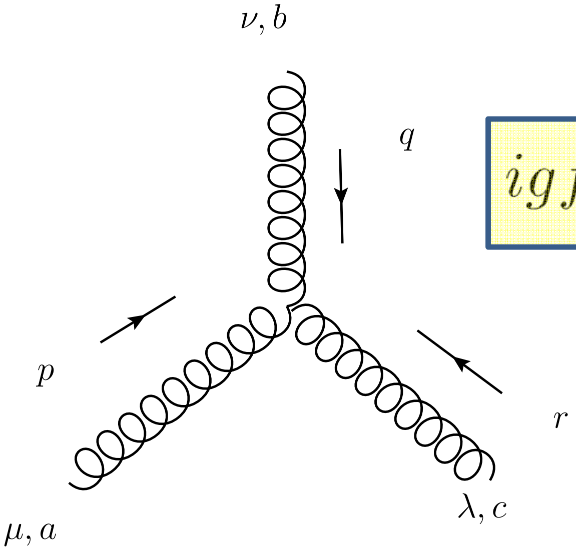


$$\frac{-i\delta_{ab} g_{\mu\nu}}{p^2}$$

(Feynman gauge)



$$i g \gamma^\mu T_a$$



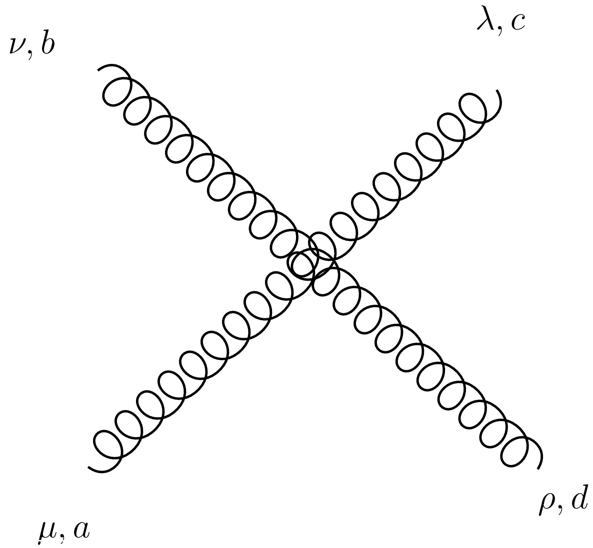
$$igf^{abc}[(p-q)_\lambda g_{\mu\nu} + (q-r)_\mu g_{\nu\lambda} + (r-p)_\nu g_{\lambda\mu}]$$

$$p+q+r=0$$

$$[T^a, T^b] = if^{abc}T^c$$

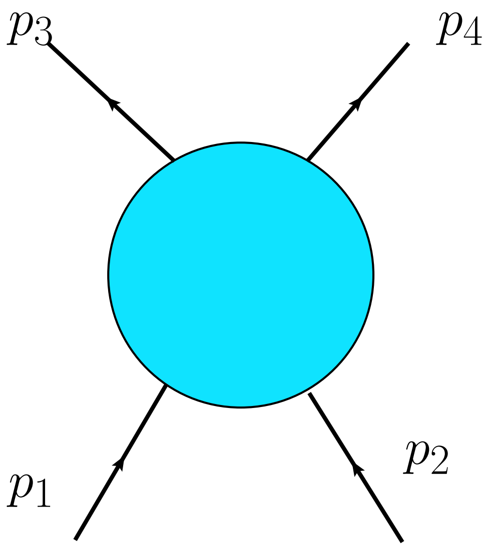
$$T^aT^a = C_F = \frac{4}{3} \qquad \mathrm{tr}\, T^aT^b = \frac{1}{2}\delta^{ab}$$

$$f^{acd}f^{bcd} = 3\delta^{ab}$$



$$-ig^2\Big[f^{abe}f^{cde}(g_{\lambda\nu}g_{\mu\rho} - g_{\lambda\rho}g_{\mu\nu})$$
$$+f^{ace}f^{bde}(g_{\lambda\mu}g_{\nu\rho} - g_{\lambda\rho}g_{\mu\nu})$$
$$+f^{ade}f^{bce}(g_{\lambda\mu}g_{\nu\rho} - g_{\lambda\nu}g_{\mu\rho})\Big]$$

Kinematics



Momentum conservation

$$p_1 + p_2 = p_3 + p_4$$

Mandelstam variables

$$\begin{aligned} s &= (p_1 + p_2)^2 = (p_3 + p_4)^2 = 2p_1 \cdot p_2 = 2p_3 \cdot p_4, \\ t &= (p_1 - p_3)^2 = (p_4 - p_2)^2 = -2p_1 \cdot p_3 = -2p_2 \cdot p_4, \\ u &= (p_1 - p_4)^2 = (p_2 - p_3)^2 = -2p_1 \cdot p_4 = -2p_2 \cdot p_3. \end{aligned}$$

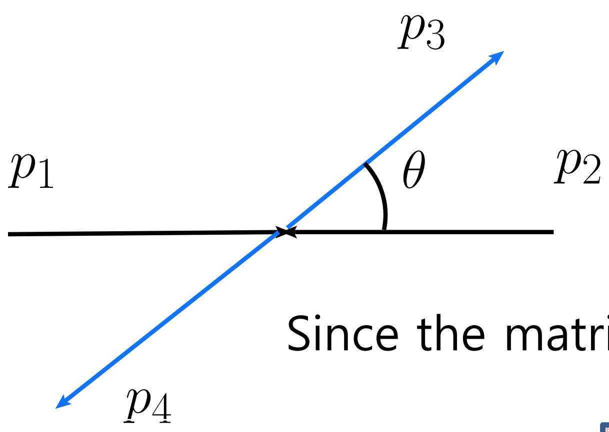
All the scattering cross sections are functions of s, t and u .

$$s + t + u = 0$$

Phase space

Scattering cross section for n final-state particles

$$d\sigma = \frac{1}{2s}\sum |\mathcal{M}|^2 (2\pi)^4 \delta^4\left(p_1 + p_2 - \sum_{i=1}^{n} p_i\right) \prod_{i=1}^{n} \frac{d^3 p_i}{(2\pi)^3 2E_i}$$



In the CM frame,

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s}|\mathcal{M}|^2$$

HW problem 1
Derive this.

Since the matrix element is independent of the azimuthal angle,

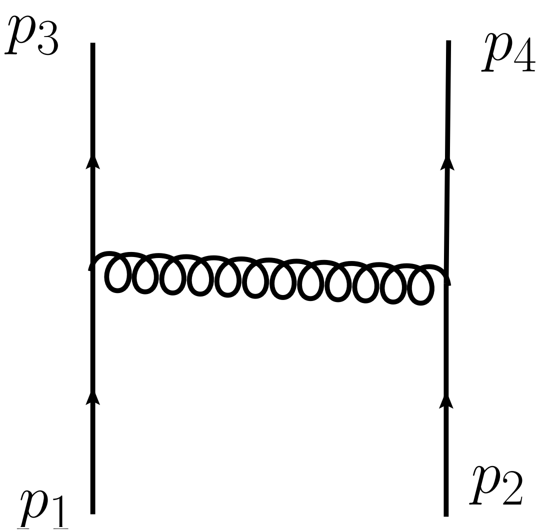
$$\frac{d\sigma}{dt} = \frac{|\mathcal{M}|^2}{16\pi s^2}$$

Homework problem 2
Derive this.

Lecture 2:
2 →2 parton processes

- We consider scattering at the parton level.
- We average over initial spin and color.

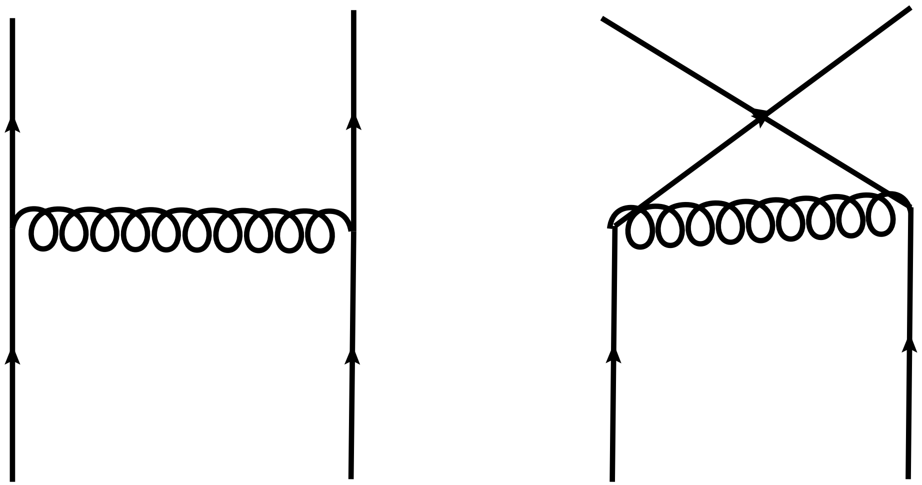
$$qq' \to qq', q\overline{q'} \to q\overline{q'}$$



$$\overline{|\mathcal{M}|^2} = g^4 \frac{4(s^2+u^2)}{9t^2}$$

$$\frac{d\sigma}{dt} = \frac{4\pi\alpha_s^2}{9} \frac{s^2+u^2}{s^2t^2}$$

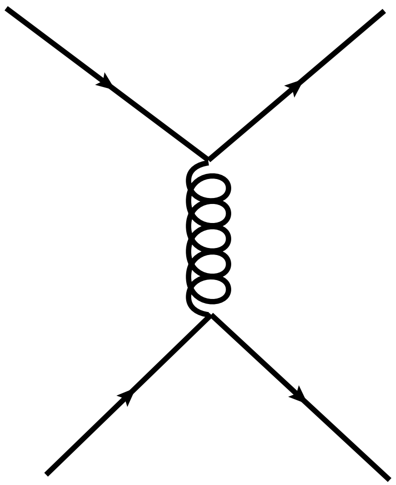
$$qq \to qq$$



$$\overline{|\mathcal{M}|^2} = \frac{4g^4}{9}\left(\frac{s^2+u^2}{t^2} + \frac{s^2+t^2}{u^2}\right) - \frac{8g^4}{27}\frac{s^2}{ut}$$

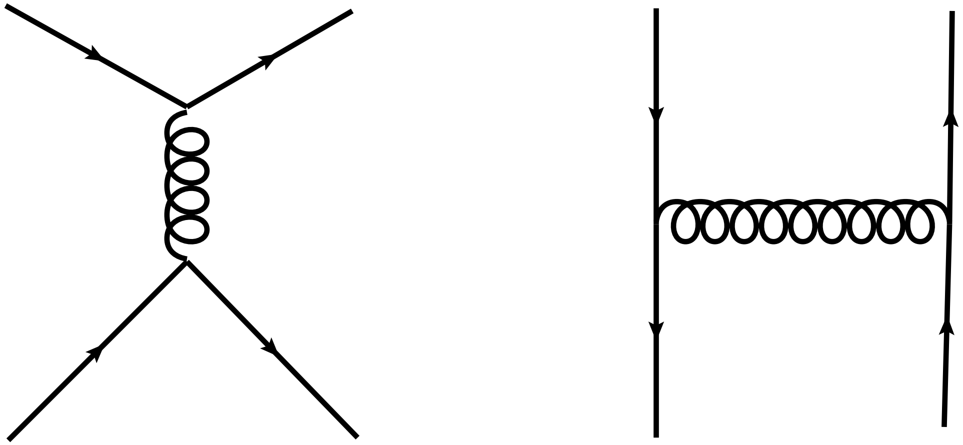
Homework Problem 3

$$q\bar{q} \to q'\overline{q'}$$



$$\overline{|\mathcal{M}|^2} = \frac{4g^4}{9}\frac{t^2+u^2}{s^2}$$

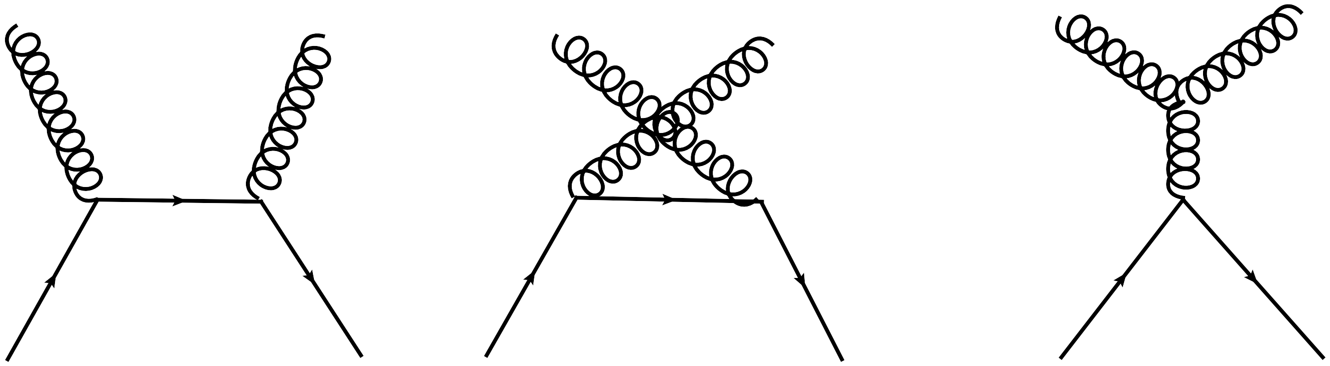
$q\overline{q} \to q\overline{q}$



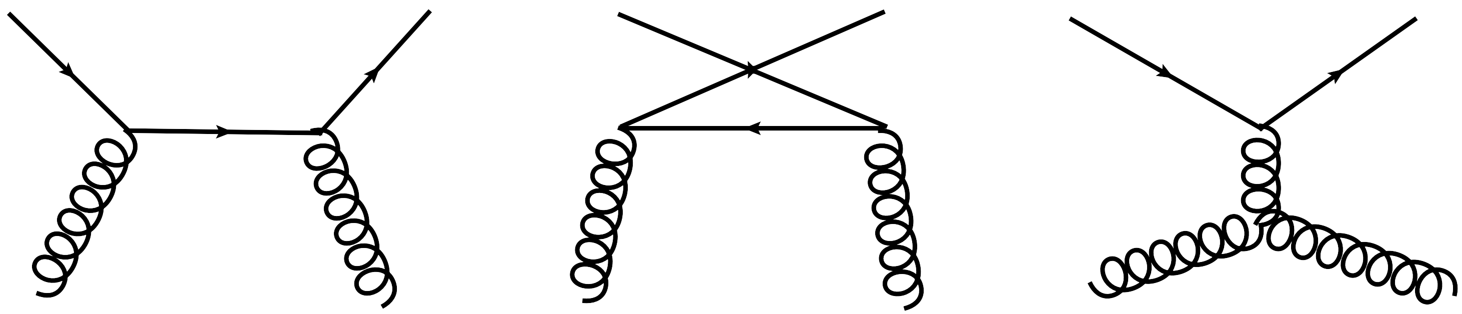
$$\overline{|\mathcal{M}|^2} = \frac{4g^4}{9}\left(\frac{s^2+u^2}{t^2} + \frac{t^2+u^2}{s^2}\right) - \frac{8g^4}{27}\frac{u^2}{st}$$

Homework problem 4

$q\overline{q} \rightarrow gg$



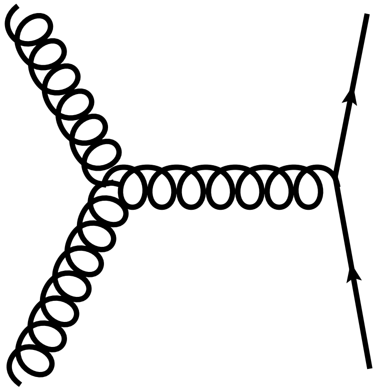
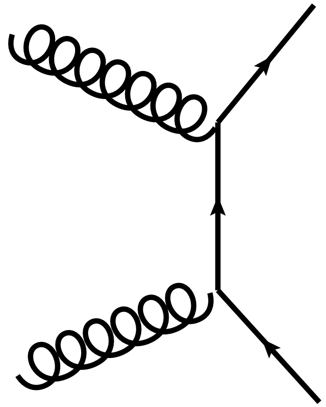
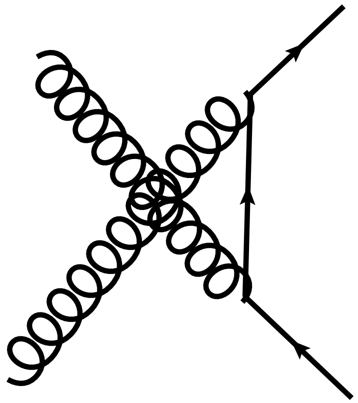
$$gg \to q\overline{q}$$



$$\overline{|\mathcal{M}|^2} = \frac{g^4}{6}\frac{u^2+t^2}{ut} - \frac{3g^4}{8}\frac{u^2+t^2}{s^2}$$

Homework problem 5

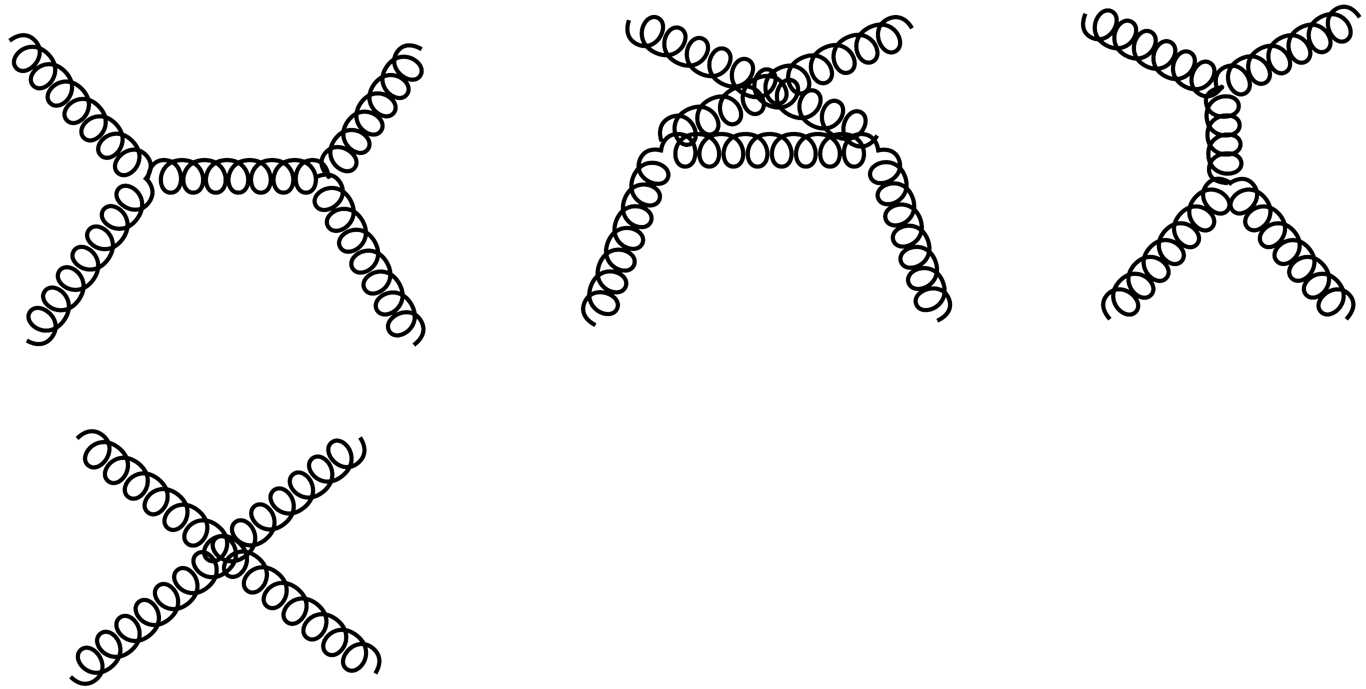
$$qg \to qg$$

$$\overline{|\mathcal{M}|^2} = g^4 \frac{s^2+u^2}{t^2} - \frac{4g^4}{9}\frac{s^2+u^2}{us}$$

Homework problem 6

$$gg \rightarrow gg$$



Challenge problem

$$\overline{|\mathcal{M}|^2} = \frac{9g^4}{4}\Big(\frac{s^2+u^2}{t^2} + \frac{s^2+t^2}{u^2} + \frac{u^2+t^2}{s^2} + 3\Big)$$